The Effect of Base Temperature Variation on The Unsteady State Annular Heat Sink

Ali Meer Ali Jasim
Babylon University / College of Engineering
Mechanical Engineering Department

Rehab Noor Mohammed
Babylon University / College of Engineering
Mechanical Engineering Department

ABSTRACT

The effect of base temperature variation on the heat transfer from unsteady state annular heat sink of cooling microelectronic device. Three types of the base temperature variation equations are taken in the present study. The Sine wave variation, Cosine wave variation & exponential variation of the base temperature. The finite element technique based on Galerkin method with axisymmetric rectangular elements is used in the present analysis. The base temperature variation effects on the effectiveness & the efficiency of the heat sink are studied during the present research. We were taken the base temperature is depends on the time & the height of the heat sink. A Quick Basic computer program were performed on a high performance PC & some typical results are plotted in graphical forms. These plots give the effectiveness & efficiency of annular heat sink as a function of the dimensionless time (Fourier no.) with different forms of base temperature. The exponential equation for the base temperature along for heat sink height will produce high temperature peak value compared with the others (Sine and Cosine), while Cosine wave produce high amplitude than Sine wave for the heat sink height.

الخلاصة

لقد تم دراسة تغير درجةِ الحرارة القاعدة على انتقال الحراره الغير مستقرةِ خلال مستقبل حراري (heat sink) محيطي الشكل. في الدراسةِ الحاليةِ, تم دراسة ثلاثة أنواعِ من معادلاتِ درجة حرارة القاعده. دالة موجةِ الجيبَ و موجةِ الجيبِ تمام وكذلك الدالة الاسية اخذت خلال تغير درجةِ الحرارة القاعدة. استخدمت خلال البحث طريقة العناصر المحدده (Finite Elements) بالاعتماد على طريقة (Galerkin) مع استخدام عناصر مستطيله و متناظره. تم دراسة تاثير تغير المستقبل الحراري على كل من الكفاءه و الفاعلية المستقبل الحراري ، حيث تم اعتبار درجة حراره القاعده تتناسب مع الزمن و وارتفاع المستقبل الحراري. لقد تم كتابة برنامج بلغة (Quick Basic) و باستعمال حاسبة ذات كفاءه عالية ، بعض النتائج تم تنظيمها و تحويلها الى مخططات و رسوم. هذه المخططات توضح فاعلية و كفاءه المستقبل الحراري مع الوقت و تغير درجة حرارة القاعده. الدالة الاسية لتغير درجةِ الحرارة القاعدة على ارتفاع المستقبل الحراري تنتج اعلى قيمة لدرجةِ الحرارة مقارنتاً مع دالة موجةِ الجيبَ و موجةِ الجيبِ تمام , بينما دالة موجةِ الجيبِ تنتج اعلى قيمة لدرجةِ الحرارة مقارنتاً مع دالة موجةِ الجيبَ.

NOMENCLATURES					
Symbol	**Description**	**Unit**	**Symbol**	**Description**	**Unit**
A	Cross section area	m^2	[C]	Elemental capacity matrix	-
k	Thermal conductivity	W/m°C	[N]	Shape function vector	-
T	Temperature	K	[K]	Elemental stiffness matrix	-
t	Time	Sec	[N*]	Unsteady shape function vector	-
Z	Longitudinal coordinate (Z- axis)	m	{R}	Residual vector	-
r	Longitudinal coordinate (r- axis)	m	[W]	Weighted function	-
S	Local coordinate in longitudinal direction	m	[Km]	Elemental stiffness matrix due to boundary condition	-
h	The average heat transfer coefficient	W/m^2 °C	L	Extended surface length	m
C	Heat Capacity		q	Heat transfer rate	W
Greek Symbols					
α	Thermal diffusivity	m^2/s	η	Overall efficiency of extended surface	-
ε	Extended surface effectiveness	-	ρ	Density	Kg/m^3
Subscript					
c	Average value between two adjustment values		o	Initial	

INTRODUCTION

Annular heat sinks are used extensively in heat exchanger devices to increase the heat transfer rate from heat source for a given temperature difference or to decrease the temperature difference between the heat source & the heat sink for a given heat flow rate. The interesting of used annular heat sinks in many thermal engineering field, such as cooling components in microelectronic package, air conditioning heat exchanger, where using heat sink, thermal analysts have succeeded in designing more compact & efficient heat transfer system. A schematic of the annular heat sink that used in the present study is shown in figure (1) in horizontal & vertical orientation. The heat sink consists of (N_f) identical fins with a centrally located circular support cylinder. Each fin has a fin thickness (t_f), an outer radius (R_o), and an inner radius (R_i). Adjacent fins are separated by a distance (b_f) & the overall length of the heat sink is (L).

Convection heat transfer from annular heat sinks can be found in the open literature. Several solutions to the problem of steady condition within in annular fin of constant thickness have been presented [1] & [2]. Edwards and Chaddock, [3], discussed the heat transfer from annular heat sink experimentally. M.M.Yovanovich [4], used a simplified solution for annular fin of constant thickness with boundary conditions of third kind (Robin condition) applied to the fin base, sides & end. Sushanta K. Mitra, [5], developed a simplified & accurate technique by means of resistance method for determining the fin resistance of constant thickness annular fin with base contact resistance & end cooling. C.S.Wang, [6], studied the natural convection heat transfer from horizontal isothermal annular heat sink, he treat the inner & outer surface separately and establish the general model which account for the effects of all surfaces of the heat sink..

The objective of the present research is study the heat transfer from annular heat sink of cooling microelectronic system by using finite element techniques. The variation of base temperature is studied here & we assumed base temperature is varied with time & the heat sink height. Then we checked the traditional assumption of the heat sink of constant base temperature.

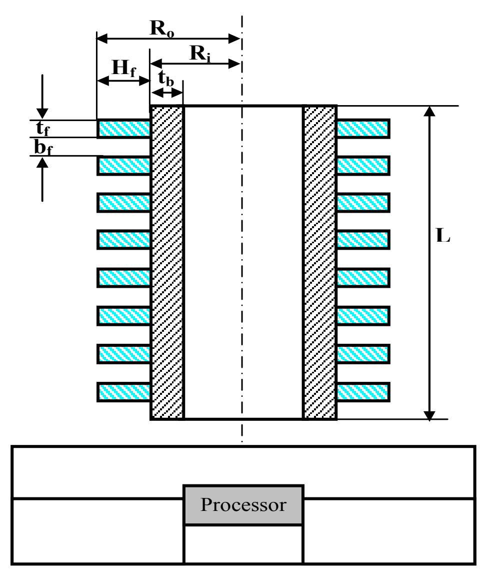

Figure (1): the annular heat sink for PC

MATHEMATICAL MODELLING

Consider constant thickness annular heat sink, as shown schematically in figure (1) .The analysis is based the following assumptions:

All the physical properties for the annular heat sink materials are assumed to be constant.

There is a perfect contact between the wall and the extended surfaces.

The environment heat transfer coefficient (h) is proportional with the film temperature (the average temperature between the fluid & surface temperatures).

The differential equation for the transient temperature distribution is formulated from a consideration of heat balance over the differential element (Δz, Δr), taken into account the assumptions of the problem. Hence, the differential equation for a two dimensional unsteady-state (transient) heat flows can be written as following:

$$\frac{\partial^2 T}{\partial Z^2}+\frac{1}{r}\frac{\partial^2 (rT)}{\partial r^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \ldots\ldots(1)$$

The Initial & boundary conditions for the case study as following:

The initial condition is given by

$$T_{initial}=T(r,Z,0)=T_{base} \qquad \text{at} \qquad t=0 \quad \ldots\ldots(2)$$

Because of symmetry of boundary conditions of the third kind which are imposed over the fin base, sides & end , then the boundary conditions as following:

At $Z = 0$ & $(R_i - t_b) < r < R_i$ $\quad k\frac{\partial T}{\partial Z}=0 \quad \ldots\ldots(3)$

At $Z = L$ & $(R_i - t_b) < r < R_i$ $\quad -k\frac{\partial T}{\partial Z}=h(T-T_f) \quad \ldots\ldots(4)$

At $r = R_o$ & $0 < Z < L$ (fin tip) $\quad -k\frac{\partial T}{\partial r}=h(T-T_f) \quad \ldots\ldots(5)$

At $R_i < r < R_o$ & $0 < Z < L$ (upper & lower surface of fins) $\quad -k\frac{\partial T}{\partial r}=h(T-T_f) \quad \ldots\ldots(6)$

At $r = R_i$ & $0 < Z < L$ (space between fins) $\quad -k\frac{\partial T}{\partial r}=h(T_b-T_f) \quad \ldots\ldots(7)$

At $r = R_i$ & $0 < Z < L$ $\quad T = T_{base} \quad \ldots\ldots(8)$

In our study, we have taken the T_{base} variable as following, [7]:

$$\left\{\begin{matrix} T_{base}=T_b+(T_b-T_f)\,\mathrm{Sin}(\frac{2\pi}{L}Z)\,\mathrm{Cos}(\omega t) \\ T_{base}=T_b+(T_b-T_f)\,\mathrm{Cos}(\frac{2\pi}{L}Z)\,\mathrm{Cos}(\omega t) \\ T_{base}=T_b+(T_b-T_f)\,\mathrm{Exp}(\frac{Z}{L})\,\mathrm{Cos}(\omega t) \end{matrix}\right\} \quad \ldots\ldots(9)$$

GALERKIN FINITE ELEMENT TECHNIQUES

To calculate the efficiency & the effectiveness, firstly, we must calculate the temperature distribution throughout the annular heat sink. The weighted residual finite element based on Galerkin method with axisymmetric rectangular elements had been used to solve the governing differential equations that mentioned previously compound with the initial & boundary conditions.

The solution of the set of the differential equations and boundary conditions established is approximated by following integral, [8]

$$\int_v W_i R dv = 0 \quad \text{……………………………………………………(10)}$$

$$R = \alpha\left(\frac{\partial^2 T}{\partial Z^2} + \frac{1}{r}\frac{\partial^2 (rT)}{\partial r^2}\right) - \frac{\partial T}{\partial t} \neq 0 \quad \text{……………………………………………(11)}$$

It is noted the above equation is not equal zero, since the approximate solution does not satisfy Fourier equation exactly

The temperature is approximated by a continuous function of Z & r. After some mathematical manipulation, it can show the temperature in a given elements as [9]:

$$T^{(e)} = \{N\}^T \{T_n\} \quad \text{……………………………………………………(12)}$$

$$T = N_i T_i + N_j T_j + N_k T_k + N_m T_m \quad \text{……………………………………………(13)}$$

Where the T_n is the nodal temperatures vector including the temperatures at the nodes attached to the element, & $\{N\}^T$ is the row factor of the element shape functions.

Where the axisymmetric rectangular elements shape function (figure (2)) as following:

$$\left\{\begin{array}{l} N_i = \frac{1}{4ab}\left(R_j - r\right)\left(Z_m - Z\right) \\ N_j = \frac{1}{4ab}\left(r - R_i\right)\left(Z_m - Z\right) \\ N_k = \frac{1}{4ab}\left(r - R_i\right)\left(Z - Z_i\right) \\ N_m = \frac{1}{4ab}\left(R_j - r\right)\left(Z - Z_i\right) \end{array}\right\} \quad \text{……………………………………………(14)}$$

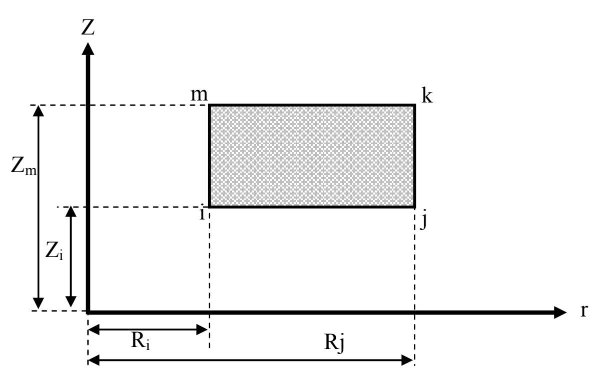

Figure (2): axisymmetric rectangular element

The residual vector of Galerkin finite element as following:

$$\left\{R^{(e)}\right\} = -\int_A [N]^T \left[\alpha\left(\frac{\partial^2 T}{\partial Z^2} + \frac{1}{r}\frac{\partial^2 (rT)}{\partial r^2}\right) - \frac{\partial T}{\partial t}\right] dA \quad \text{……………………………………(15)}$$

$$\left\{R^{(e)}\right\} = -\int_A [N]^T \alpha \left(\frac{\partial^2 T}{\partial Z^2} + \frac{1}{r}\frac{\partial^2 (rT)}{\partial r^2} \right) dA + -\int_A [N]^T \frac{\partial T}{\partial t}\, dA \quad \text{………………..............…(16)}$$

With the substitution of eq. (3) in eq. (1) and application the Galerkin method weighted residual statement can be rewritten as:

$$[C]\left\{\frac{\partial\{T\}}{\partial t}\right\} + [k]\{T\} + \{F\} = 0 \quad \text{..……………...………………………………………..(17)}$$

In the above equation, the element distribution of the global capacitance matrix [C], of the global conductivity matrix [k], and the global force vector {f} are:

$$[C^{(e)}] = \int_{Vol} \rho C_p r [N][N]^T d(vol) \quad \text{……..…………………...…………...……………..(18)}$$

$$\left[k^{(e)}\right] = \int_{Vol} [B][D][B]^T d(vol) + \int_{Surf} h [N][N]^T d(surf) \quad \text{..…………………………..(19)}$$

$$\left\{f^{(e)}\right\} = \int_{Surf} h T_f [N]^T d(surf) \quad \text{..……………………………………..…………..(20)}$$

Where

$$[B] = \begin{bmatrix} \dfrac{\partial [N]}{\partial Z} \\ \dfrac{\partial [N]}{\partial r} \end{bmatrix} \qquad \text{And} \qquad [D] = \begin{bmatrix} k_{zz} & 0 \\ 0 & k_{rr} \end{bmatrix} \quad \text{……………………….……...…(21)}$$

$$\left[K^{(e)}\right] = \left[K_M^{(e)}\right]\{r\} + \left[K_s^{(e)}\right]\{r\} \quad \text{………………………...….………………….…….…...…(22)}$$

Where

$$\left[K_M^{(e)}\right] = \frac{2\pi\bar{r}ak}{6b}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{2\pi\bar{r}ak}{6b}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix} \quad \text{……………………..(23)}$$

$$\left[K_s^{(e)}\right] = \frac{2\pi h L_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{…………..………………………………………..(24)}$$

$$\{r\} = \begin{bmatrix} R_i \\ R_j \\ R_k \\ R_m \end{bmatrix} \quad \text{……………………………..………..……………………………..(25)}$$

$$\left\{f_s^{(e)}\right\} = \frac{2\pi h T_f L_{ij}}{6}\begin{bmatrix} 2R_i + R_j \\ R_i + 2R_j \\ 0 \\ 0 \end{bmatrix} \quad \text{…………..……………………….……………...(26)}$$

$$\left[c^{(e)}\right]=\frac{2\pi\bar{r}a\rho C_p}{4b}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \quad \ldots\ldots(27)$$

Where, the (L_{ij}) is the length of the respective side of the element that subject to the boundary condition & the ($\bar{r}$) is the average radius of the elements . There are three other results of $\left[K_s^{(e)}\right]$& $\left\{f_s^{(e)}\right\}$, one for each element side.

The linear first- order differential equations in the time domain will produce in finite element solution. These equations must be solved before the variation of (T) in space and time. A several procedures can be used for numerically solving of the eq. (25).

Give a function T(t) and the interval [a, b], we can use the mean value theorem for differentiation to develop an equation for T(t). Then the final equation for temperature with time is:

$$\{T\}=(1-\theta)\{T\}_a+\theta\{T\}_b \quad \ldots\ldots(28)$$

And after substitution the above equation in equation (13), will produce:

$$([C]+\theta\,\Delta t\,[k])\{T\}_{t+\Delta t}=([C]-\theta\,\Delta t\,[k])\{T\}_t+((1-\theta)\{f\}_t+\theta\{f\}_{t+\Delta t}) \quad \ldots\ldots(29)$$

In above equation, θ = 0; ½ & 1 corresponding to the explicit, Crank-Nicholson and purely implicit solution to the system, in our study we take (θ=1/2) (Crank Nicholson).

The convective heat transfer coefficient formula in the present study as following [10]:

$$h_r=7.2\,V_r^{0.78} \quad \ldots\ldots(30)$$

Where Vr is the air velocity along the plate,

Since mass velocity is the fundamental variable in forced-convection equations, Schack recommends that the velocity should be corrected according to the ideal gas law for temperatures. The velocity along the plate can be expressed by

$$V=\frac{\dot{m}}{A_{flow}\,\rho} \quad \ldots\ldots(31)$$

Substituting Eq. 31 into Eq. 30 gives the convective heat transfer coefficient as

$$h=7.2\left(\frac{\dot{m}\,R\,T_{film}}{A_{flow}\,\rho}\right)^{0.78} \quad \ldots\ldots(32)$$

Where T_{film} is the film temperature in Kelvin

The total heat lost from the heat sink heat sink equal to the summation of heat lost from each element which subjected to the boundary condition (air) as following:

$$q_{ct}=\sum_{i=1}^{BC}h_i\,A_i\left(T_i-T_f\right) \quad \ldots\ldots(33)$$

$$A_c=2\,\pi\,i\,\Delta r\,.\,\Delta Z \quad \ldots\ldots(34)$$

The ratio of the actual heat lost from heat sink to the heat lost from the bared cylinder surface without fins is called heat sink effectiveness, it is calculated as following:

$$q_{ct}=\sum_{i=1}^{BC}h_i\,A_i\left(T_i-T_f\right) \quad \ldots\ldots(35)$$

$$A_o=2\,\pi\,r_i\,.\,L \quad \ldots\ldots(36)$$

$$h_o = 7.2\left(\frac{\dot{m}\,R\,T_{av}}{A_{flow}\,\rho}\right)^{0.78} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(37)$$

Then, the heat sink effectiveness

$$\varepsilon = \frac{\sum_{i=1}^{BC} h_i\ i\,\Delta r\,\Delta Z\left(T_i - T_f\right)}{h_o\, r_i\, L\left(T_{base} - T_f\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(38)$$

The ratio of actual heat transfer lost by heat sink to the heat lost from the same heat sink but it's surfaces temperature are constant and equal to base temperature, as following

$$q_{fb} = \sum_{c=1}^{BC} h_b\ A_i\left(T_b - T_f\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(39)$$

Then, the heat sink efficiency

$$\eta = \frac{\sum_{i=1}^{BC} h_i\ i\,\Delta r\,\Delta Z\left(T_i - T_f\right)}{\sum_{i=1}^{BC} h_b\ i\,\Delta r\,\Delta Z\left(T_b - T_f\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(40)$$

The nodals temperature are assumed to be converged to the steady-state values when the difference of the nodals temperature between two alternative time steps (n) and (n+1) satisfies the equation

$$\text{Error} = \left|\left(T_i^{j+1} - T_i^{j}\right) / \ T_i^{j}\right| \le 10^{-6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(41)$$

RESULTS & DISCUSSION

Before the discussion of the results, we must check the accuracy of the present program & analysis, and then we compared the results of the present program with the two dimensional heat transfer from annular fins, [4] in figure (3). For two cases, when the heat from the extended tip insulated & non-insulated. From this figure, we can observe a good accuracy in our program & technique.

Also, we can showed the variation of the heat sink efficiency with the Biot number, the efficiency decreased as the Biot number increased due to increasing of the heat transfer coefficient, the increasing of heat transfer coefficient will increase the heat transfer removed from the heat sink & then reduce the temperature difference between the surface & the environment temperatures with constant base temperature.

Figures (4) & (5), showed the efficiency & the effectiveness of the heat sink at a constant Biot number & variable base temperature. Because of we took the initial temperature equal to the base temperature in our case study, the efficiency & the effectiveness of heat sink start from the high value & then its decreased as the time (Fourier number) increased to reach the steady state value, after this value no change will occurs in the efficiency & the effectiveness of the heat sink with time (Fourier umber) increased. In our steady we took three cases for the variation of the base temperature with engine overall length (Sine, Cosine, and Exponential) & with time effect took the cosine wave form. We can note the exponential form for engine length will produce high temperature peak value compared with the others, while cosine wave produce high amplitude than sine wave for the engine length.

In order to check the effect of the variation of the base temperature on the efficiency & effectiveness of the annular heat sink, we took different forms for base temperature.

As shown in figures (6) & (7), the periodic cosine time variation is replaced with sine time variation (see the legends of figures (6) & (7)), we can show nearly the same behaviors of the previous figures (4) & (5), also the base temperature for the exponential form will produce high amplitude.

For special case, where the base temperature of the annular heat sink changed exponentially with time (see the legends of figures (8) & (9)), we can note the efficiency & the effectiveness for this case not reach to the steady state value & the efficiency reduced than steady state value as the time (Fourier number) increased while the effectiveness of the annular heat sink firstly decreased to reach the steady state value & then increased sharply after this value, this behavior due to the base temperature reach to the large value abnormal distribution as the time increased.

CONCLUSION

The unsteady heat transfer from annular heat sink of cooling microelectronic device are solved numerically here by using finite elements (Galerkin method) with axisymmetric rectangular elements. The results of the present work showed the large effect of the base temperature on the efficiency & the effectiveness of annular heat sink. The exponential time change of base temperature is not produced & reach to the steady state value, and the exponential engine height change of base temperature for other two cases will produce high temperature peak & variation in the base temperature..

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List of Figures

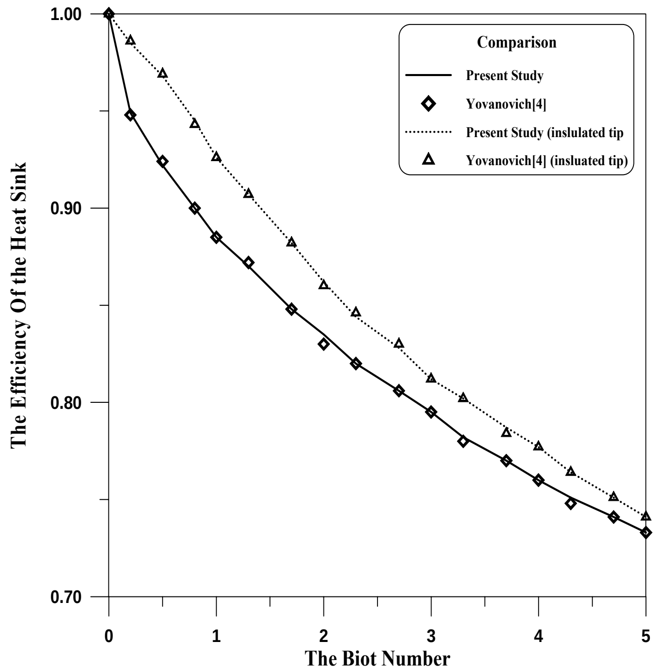

Figure (3): show the comparison for two cases extended insulated tip & non insulated tip

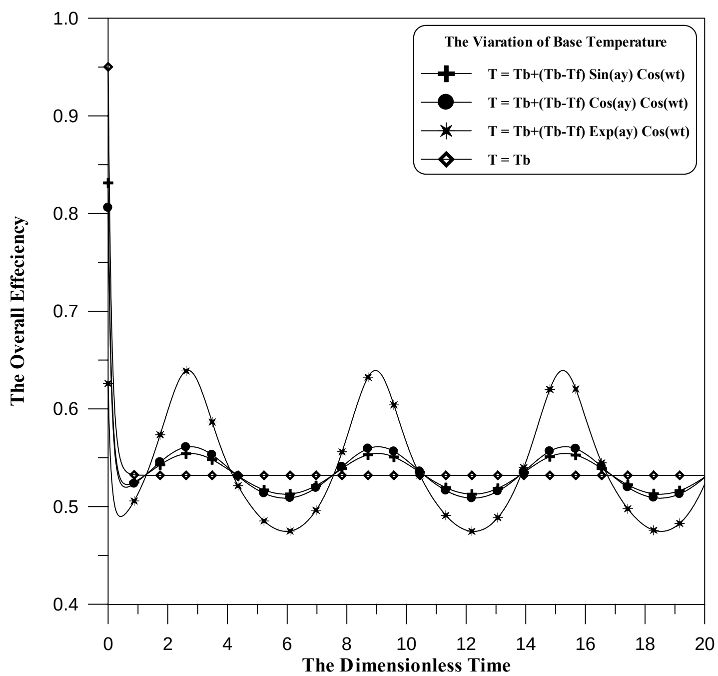

Figure (4): show the efficiency against the dimensionless time

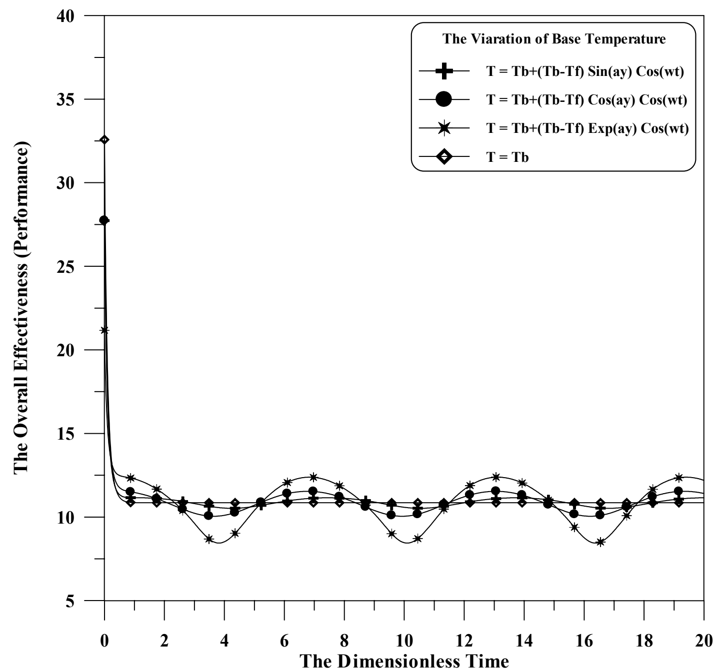

Figure (5): show the effectiveness against the dimensionless time

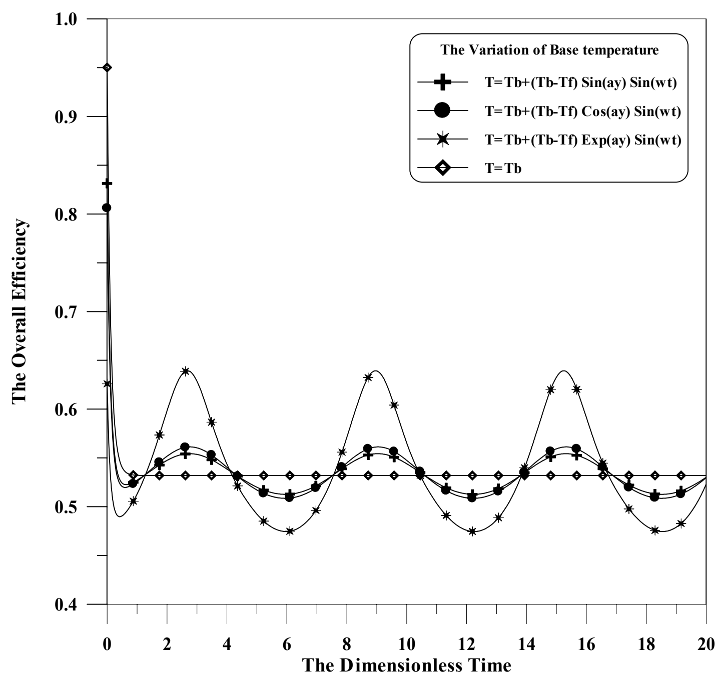

Figure (6): show the efficiency against the dimensionless time

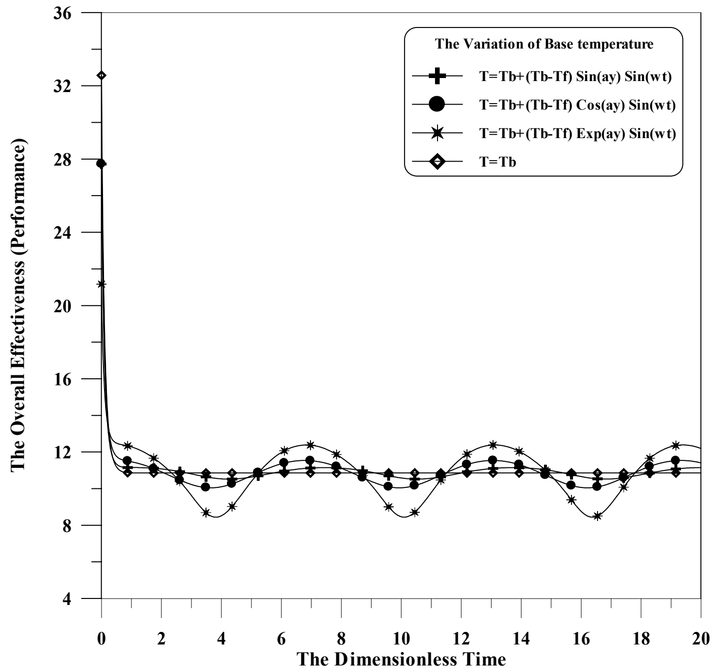

Figure (7): show the effectiveness against the dimensionless time

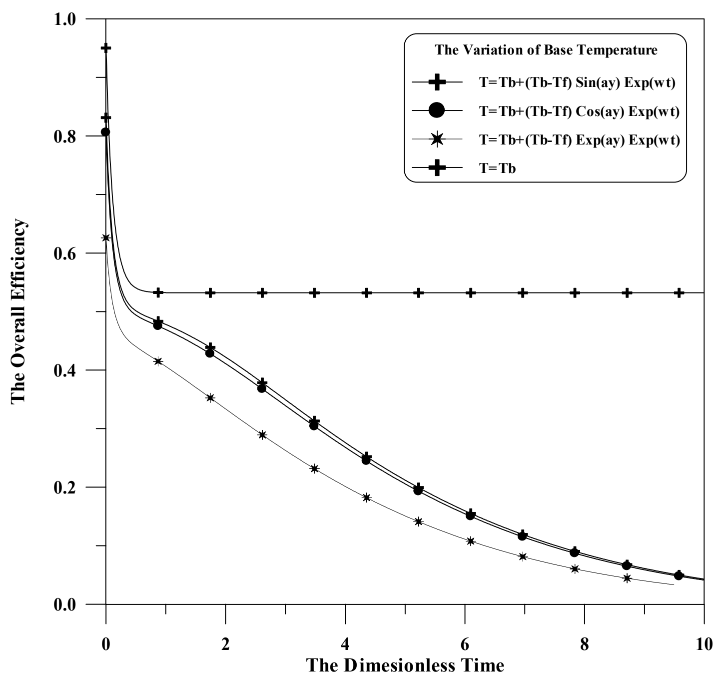

Figure (8): show the efficiency against the dimensionless time

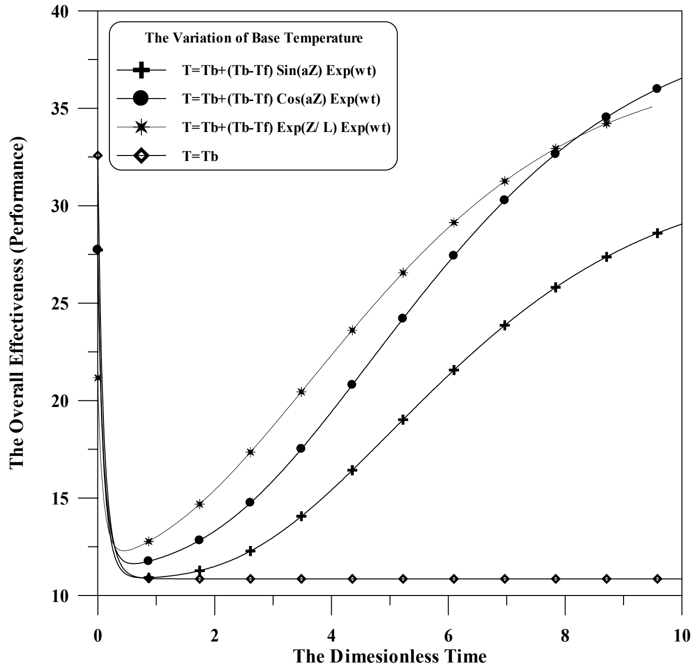

Figure (9): show the effectiveness against the dimensionless time